Math 342: Homework 4

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Documentation: I used ChatGPT solely for looking up Latex commands. The main Homework 4 MatLab script and all required dependencies are located in the Homework 4 folder found here: <https://github.com/Connor-Lemons/Emmons-Math-342>. No other resources used.

Problem 1 (5c, 7c):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | approx | Error bound | Actual Error |
| 2.9 | -4.827866 | 5.1014 | 0.0181 | 0.012 |
| 3.0 | -4.240058 | 6.6548 | 0.0090 | 0.0049 |
| 3.1 | -3.496909 | 8.2163 | 0.0049 | 0.00048 |
| 3.2 | -2.596792 | 9.786 | 0.0099 | 0.0014 |

All work can be found in MatLab script attached at the end of the document, or in the GitHub page.

Problem 2 (29):

Consider the function which describes total error . The minimum of this function will occur when its derivative is equal to zero, which gives:

(1)

Rearranging gives:

(2)

Thus the error function will be minimized at this value of h.

Problem 3 (15c):

Approximate using closed Newton-Cotes Formulas up to and open Newton-Cotes Formulas up to .

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Trapezoid | Simpson | Simpson Three-Eights | Closed | Midpoint | Open | Open | Open |
| Value | 1.497171 | 1.477536 | 1.477529 | 1.477523 | 1.467719 | 1.470981 | 1.477512 | 1.477515 |
| Error  Bound | 0.0239 | 1.59e-5 | 7.08e-6 | 3.79e-9 | 0.0120 | 0.00797 | 1.39e-5 | 9.69e-6 |
| Actual Error | 0.0196 | 1.31e-5 | 5.81e-6 | 3.11e-9 | 0.00980 | 0.00654 | 1.14e-5 | 7.95e-6 |

The actual error of each of these methods is within the error bound for each of these methods. The most accurate method is the closed Newton-Cotes with . All work can be found in MatLab script attached at the end of the document, or in the GitHub page.

Problem 4 (13b):

Approximate within using composite Simpson’s rule.

For the approximation to be within , begin with:

(1)

Substituting and for the endpoints gives:

(2)

Note that the fourth derivative of is , which gives:

(3)

The derivative is maximized at with a value , which gives:

(4)

Solving for gives that , which gives by . Note that this means that , which is the smallest even integer that satisfies the given condition.

Implementing Composite Simpson’s Rule with gives the following approximation:

(5)

MatLab code can be found in the GitHub page or at the end of the document.

Problem 5 (1b, 3b):

Compute the Simpson’s rule approximations , , and for .

(1)

(2)

(3)

Consider Simpson’s rule for some integral:

(4)

Applying Composite Simpson’s Rule with and gives:

(5)

Letting and simplifies equation (5) to:

(6)

Assuming that for Simpson’s method and for Composite Simpson’s Method are approximately equal (and therefore the value of the fourth derivative of the function evaluated at is also relative equal), then:

(7)

Simplifying gives:

(8)

Using this estimate in conjunction with equation (6) gives:

(9)

Let . This means that, for , . Computing this gives:

(9)

Because the inequality holds, is assumed to be a good approximation for .

Problem 6 (13):

Consider the Trapezoid Rule for some integral:

(1)

Additionally, consider the Composite Trapezoid Rule for the same integral:

(2)

Letting and gives:

(3)

Assuming that gives:

(4)

Rearranging gives:

(5)

In conjunction with equation (3), this gives:

(6)

Problem 7 (2a, 4a):

Approximate using Gaussian quadrature. Transforming this integral so that Gaussian quadrature may be used by gives:

(1)

(2)

To approximate this integral, find the coefficients and x-values such that:  
 (3)

This is done using the Legendre polynomials, and the coefficients and x-values are tabulated for . For the case, table 4.12 gives:

(4)

And for the case:

(5)

The error for each case is:

:

(6)

:

(7)

Going from to produces an error which is one order of magnitude smaller.

The MatLab code can be found in the GitHub page or at the back of this document.

Problem 8 (1a):

(1)

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